Factoring Polynomials

I. **Is there a greatest common factor?** GCF: shared numerical factor, variables to the smallest exponent, and/or quantity. If factorable, always factor out a negative lead coefficient.

\[ 2x - 8 = 2(x - 4) \]
\[ -x^2y^2 - 3xy^2 = -xy^2(x + 3) \]
\[ 5(x - y) + n(x - y) = (x - y)(5 + n) \]

II. **Is it a binomial?** If so, is it the...

A. **Difference of two squares?** parentheses, parentheses, plus and minus—square root both terms

\[ a^2 - b^2 = (a + b)(a - b) \]
\[ 9x^2 - 25y^2 = (3x + 5y)(3x - 5y) \]
\[ (a + b)^2 - 25 = [(a + b) + 5][(a + b) - 5] = (a + b + 5)(a + b - 5) \]

B. **Sum of two squares?** \( a^2 + b^2 \) does not factor (it is prime.)

C. **Sum of two cubes?** 1st factor—cube root both terms; 2nd factor—SOPPS (square 1st, opposite, product of terms, plus, square 2nd)

\[ a^3 + b^3 = (a + b)(a^2 - ab + b^2) \]
\[ 8x^3 + 27y^3 = (2x + 3y)(4x^2 - 6xy + 9y^2) \]

D. **Difference of two cubes?** 1st factor—cube root both terms; 2nd factor—SOPPS (square 1st, opposite, product of terms, plus, square 2nd)

\[ a^3 - b^3 = (a - b)(a^2 + ab + b^2) \]
\[ x^3 - 64 = (x - 4)(x^2 + 4x + 16) \]

E. **None of these?** It does not factor (it is prime)

III. **Is it a trinomial** \((ax^2 + bx + c)\)? If so, is it...

A. **Square of a binomial** (often referred to as a Perfect Square Trinomial)? Are the lead coefficient and c term perfect squares?

\[ a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2 \]
\[ 16x^2 + 24x + 9 = (4x + 3)(4x + 3) = (4x + 3)^2 \]
\[ 4x^2 - 20xy + 25y^2 = (2x - 5y)^2 \]

B. Is \( a = 1 \)? Use REVERSE FOIL—read backwards “factors of c that add (or subtract) to get b; SIGNS: if “sum”, signs the same; if “difference”, signs different and large one gets the sign; now break up the \( x^2 \)

\[ x^2 + 7x + 12 = (x + 3)(x + 4) \]
\[ x^2 - 7x + 12 = (x - 3)(x - 4) \]
\[ x^2 + 3x - 18 = (x + 6)(x - 3) \]
\[ x^2 - 3x - 18 = (x - 6)(x + 3) \]

C. Is \( a \neq 1 \)? Use outer/inner combinations or **AC GROUPING method.** (See handout on Factoring \( Ax^2 + Bx + C \))

**Example:** \( 2a^2 - 7a + 6 \)

Read in reverse:
1. outer and inner products that **add** to 7
2. SIGNS: if “sum”, signs the same
3. break up \( a^2 \) \((1x - 2)(2x - 3)\)

**Example:** \( 4x^2 - 8x - 5 \)

Read in reverse:
1. outer and inner products that **subtract** to get 8
2. SIGNS: if “difference”, signs different and large product (outer/inner) gets the sign
3. break up \( a^2 \) \( (2x + 1)(2x - 5) \)
**AC GROUPING method**

Example: \(2a^2 - 7a + 6\)  

AC = 2 * 6 = 12  
...factors of 12 that add to get -7

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(would add to -7)

2 3 2 3 (like number-pattern so factor by grouping)

\(2a^2 - 3a - 4a + 6\)

**IV.** Does it have four terms. If so, will it...

A. Group (first two terms together, last two terms together...look for a number pattern)

\[
\begin{array}{ccc}
5/5 & 5/5 & 1 \\
1 & 1 & 1 & 1
\end{array}
\]

\[
5ax - 5bx + a - b = (5ax - 5bx) + (a - b)
\]

\[
= 5x (a - b) + 1(a - b)
\]

\[
= (a - b)(5x + 1)
\]

\[
\begin{array}{ccc}
1 & 3 & 1 \\
3 & 1 & 3
\end{array}
\]

\[
x^3 - 3x^2 + 2x - 6 = (x^3 - 3x^2) + (2x - 6)
\]

\[
= x^2(x - 3) + 2(x - 3)
\]

\[
= (x - 3)(x^2 + 2)
\]

B. Group (first three terms together)

\[
x^2 + 6x + 9 - y^2 = (x^2 + 6x + 9) - y^2
\]

\[
= (x + 3)^2 - y^2
\]

\[
= [(x + 3) + y][(x + 3) - y]
\]

C. Group (last three terms together)

\[
y^2 - x^2 + 6x - 9 = y^2 - (x^2 - 6x + 9)
\]

\[
= y^2 - (x - 3)^2
\]

\[
= [y + (x - 3)][y - (x - 3)]
\]

**Summary of the Factoring Process**

GCF out first: What do each of the terms share
Remember to factor out a negative if the lead form is negative

Binomials

Trinomials

4 terms factor by grouping

Difference of Squares
Parentheses plus and minus
\(x^2 - y^2\)
Note: sums of squares is prime

Sum or Diff of Cubes
\(x^3 + y^3\) (SOPPS)

Id coeff of t: \(x^2 + bx + c\)
Read in reverse: 1:
factors of \(c\) that give a sum (difference) of \(b\):
2: SIGNS: sum-same; difference-different
Sign goes to the larger:
3: Break up \(x^2\)

Id coeff other than 1
\(ax^2 + bx + c\)
Outer and Inner combos