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Mathematics Success Tips Series: Implied Domains

Warning The information given here is not given in technical mathematical definitions and is not the complete knowledge that you need. Instead it attempts to give a basic understanding of the concept using non-rigorous terms.

Notice that when a function is written in function notation, such as f(x)=4x+1, the domain is not as easy to see as it is when the function is written in ordered pairs. The domain is considered implied which means we use all inputs that would give us a real answer. So to find the implied domain we think about what inputs would not give us a real answer. This list will grow as your mathematical knowledge progresses. You may only work with some of the bullet points in your course, or you may have even more restrictions that are listed here, but these are the basic domain restrictions for an algebra course.

If none of the following conditions apply the default implied domain is all real numbers, written as $(-\infty,\infty)$.

- We know that zero in the denominator of a fraction is undefined so we **exclude any inputs (x's) which would give us zero in the denominator**.

Example 13: Find the domain of $f(x)=\dfrac{3x^2+2x-1}{x-7}+5$.

Solution 13: When finding domains we only worry about the part of the problem that would result in the non-real output. So for this problem we're only concerned with the denominator so that will be the only part of the problem we'll use to find the domain. So we want $x-7\neq 0$. Then we solve this just like an equation by adding 7 to both sides to get $x\neq 7$. Many times instructors want your answer in interval notation. The interval notation for $x\neq 7$ is $(-\infty,7)\cup(7,\infty)$.

- Also negative numbers under an even root yield non-real answers so **expressions inside of even roots need to be greater than or equal to zero**.

Example 14: Find the domain of $g(x)=-2\sqrt{3x+1}-9$.

Solution 14: Again, we are only concerned about the part of the problem that would result in the non-real output. So for this problem we're only concerned with the part under the square root. So we need $3x+1\geq 0$. Then we solve this as we would any other inequality to get $x\geq -\frac{1}{3}$. The interval notation for this is $[-\frac{1}{3},\infty)$.

- Logarithms (of any base) can only take positive inputs. So **expressions inside of logs need to be greater than zero**.

Example 15: Find the domain of $f(x)=7\log(x+1)-5$.

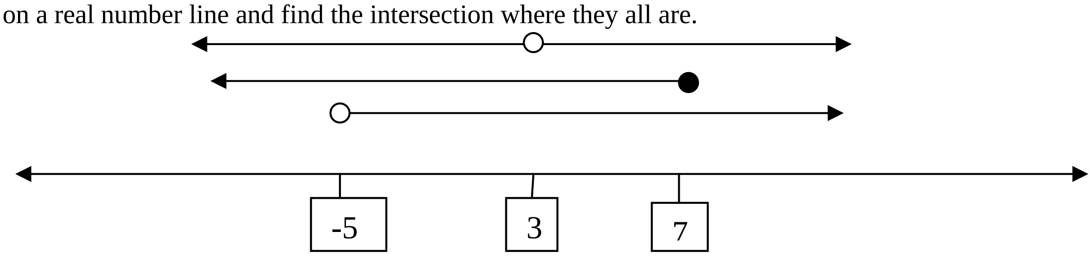
Solution 15: We need $x+1>0$. We solve the inequality to get $x>-1$ and write it in interval notation as $(-1,\infty)$.

Story problems use the implied domains from the equations that you come up with, but they also need you to use some common sense as well. When we're talking about real life physical objects for example we can't have a negative number and often not zero. These restrictions should be apparent from the wording of the problem. For example, if we have a problem where we're cutting away part of a 10 ft long string, we better not cut away more than 10 feet from it. It also wouldn't make sense to cut away a negative amount of string. So the inputs we could have for amount of string to cut away would be between 0 and 10, written $(0,10)$.

Putting it all together. Some functions may have multiple domain restrictions. To solve these more complicated problems you find each of the domain restrictions then find the interval which excludes all of the restrictions.

Example 16: Find the domain of $f(x) = \dfrac{\sqrt[4]{14-2x}}{x-3} + \ln(x+5)$.

Solution 16: First we have a denominator which we need to be non-zero so $x-3 \neq 0$ giving $x \neq 3$. Also we have an even root so we need $14-2x \geq 0$. We solve that to get $7 \geq x$. The we also have a logarithm so we need $x+5>0$ yielding $x>-5$. One of the easiest ways to see how to put all of this together is to graph each part on a real number line and find the intersection where they all are.



So the part where all three lines is between -5 and 7 but not including 3. So the final domain is $(-5,3) \cup (3,7]$.

Here are some problems for you to practice with. For all of them, find the domain of the function.

1. $f(x) = \dfrac{1}{4x^2 - x - 5} + 11$
2. $f(x) = \dfrac{7x^2 - 1}{12 - x} + \dfrac{3}{2x}$
3. $f(x) = x^2 - 3x + 9$
4. $y = \log_2(x+2) - 1$
5. $g(x) = \ln\left(\dfrac{3x+4}{2}\right) + 5$
6. $f(x) = 2\dfrac{x}{\sqrt{3x-1}}$
7. $h(x) = \dfrac{3x^2 + x}{1 - x} + 5\sqrt{x+4}$
8. $y = \dfrac{\sqrt{x^2 - 4}}{x - 7}$